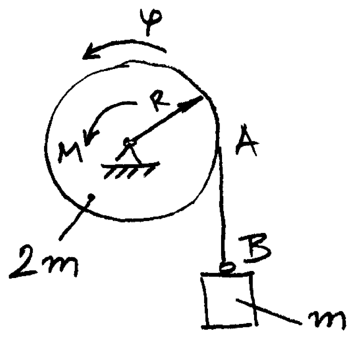
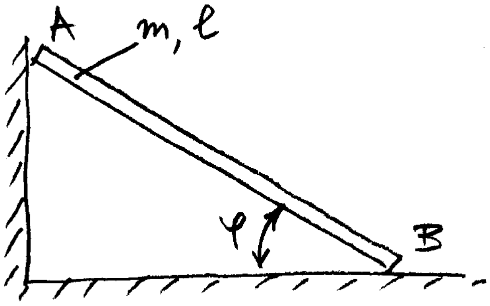
ДИНАМИКА И ОСЦИЛАЦИИ



1.) Товар со маса m се дига со јаже кое се намотува на барабан со радиус R и маса 2m. На барабанот дејствува погонски момент M.

- Да се постави диференцијалната равенка на движењето на барабанот по аголот φ (15 б.)
- Да се определи забрзувањето на товарот при константен вртежен момент M=const. (5 б.)
- Да се најде силата во јажето AB. (5 б.)

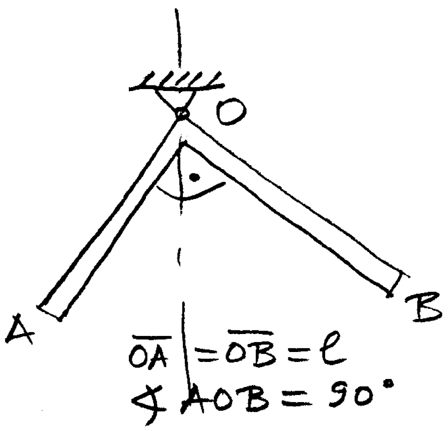


2.) Стап со должина l и маса m е пуштен да паѓа без почетна брзина од почетен агол $\varphi_o=30^o$.

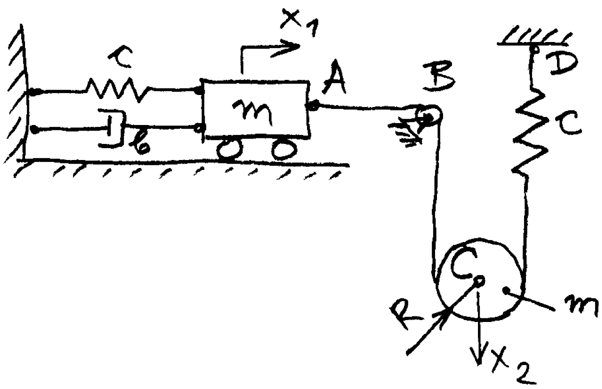
- Да се изрази аголната брзина на стапот во функција од аголот φ. (15 б.)
- Да се најде брзината на точката A од стапот непосредно пред ударот во подлогата (10 б.)

Триењето да се занемари.

Упатство. Да се применизаконот за промена на кинетичката енергија



3.) Да се најде сопствената фреквенција на малите осцилации на аголникот со маса m, кој е обесен зглобно за неговото теме O околу кој може да сениша. (20 б.)



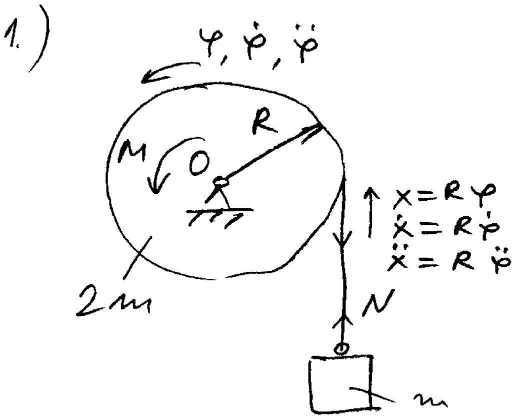
4.) Да се постави карактеристичната равенка $\Delta(\lambda)=0$ на слободните придушени осцилации на дадениот механички осцилаторен систем. Макарата C може да се движи вертикално, без бочно нишање. (30 б.)

Динамика и осцилации

02.09.2011

(1)

1.)



1 ст. на слоб., φ – генер. коорд.

$$E_K = \frac{1}{2} J_o \dot\varphi^2 + \frac{1}{2} m v_B^2 \;; \quad J_o = \frac{1}{\cancel{2}} \cancel{2} m R^2 = mR^2$$

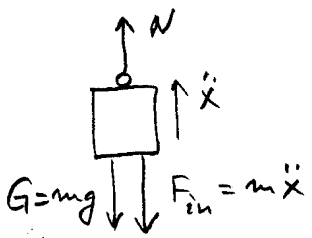
$$v_B = \dot x = R\cdot\dot\varphi \;; \quad E_K = \frac{1}{2}\cdot mR^2\dot\varphi^2 + \frac{1}{2} m (R\dot\varphi)^2$$

$$\boxed{E_K = mR^2\dot\varphi^2} \quad \boxed{E_p = mgx = mgR\varphi}$$

$$\boxed{Q_\varphi = M} \quad \frac{d}{dt}\frac{\partial E_K}{\partial\dot\varphi} - \frac{\partial E_K}{\partial\varphi} + \frac{\partial E_p}{\partial\varphi} = Q_\varphi$$

$$2mR^2\ddot\varphi + mgR = M \;/:(2mR^2) \quad \boxed{\ddot\varphi = \frac{M}{2mR^2} - \frac{g}{2R}}$$ – диф. рав.

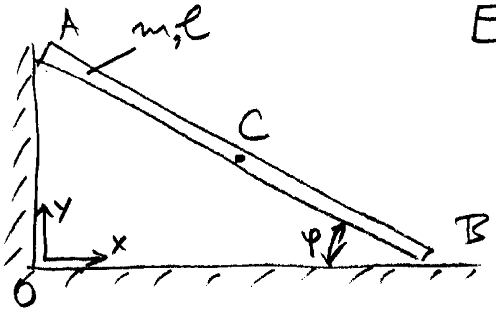
$$a = \ddot x = R\ddot\varphi = \frac{M}{2mR} - \frac{g}{2}$$



$$N - mg - m\ddot x = 0 \quad N = mg + m\left(\frac{M}{2mR} - \frac{g}{2}\right)$$

$$\boxed{N = \frac{mg}{2} + \frac{M}{2R}}$$

2.)



$$E_K = \frac{1}{2} J_c\cdot\dot\varphi^2 + \frac{1}{2} m v_c^2 \;; \quad J_c = \frac{1}{12} m\ell^2$$

$$v_c^2 = \dot x_c^2 + \dot y_c^2 \;; \quad \begin{cases} x_c = \frac{\ell}{2}\cos\varphi \\ y_c = \frac{\ell}{2}\sin\varphi \end{cases} \quad \begin{cases} \dot x_c = -\frac{\ell}{2}\sin\varphi\cdot\dot\varphi \\ \dot y_c = \frac{\ell}{2}\cos\varphi\cdot\dot\varphi \end{cases}$$

$$v_c^2 = \left(-\frac{\ell}{2}\sin\varphi\cdot\dot\varphi\right)^2 + \left(\frac{\ell}{2}\cos\varphi\cdot\dot\varphi\right)^2 = \frac{\ell^2}{4}\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1}\cdot\dot\varphi^2$$

$$v_c^2 = \frac{\ell^2}{4}\cdot\dot\varphi^2$$

$$E_K = \frac{1}{2}\cdot\frac{1}{12} m\ell^2\dot\varphi^2 + \frac{1}{2} m\frac{\ell^2}{4}\cdot\dot\varphi^2 \quad \boxed{E_K = \frac{1}{6} m\ell^2\dot\varphi^2}$$

$$\boxed{E_p = mgy_c = mg\frac{\ell}{2}\sin\varphi} \quad E_K(\varphi) + E_p(\varphi) = E_K(\varphi_0) + E_p(\varphi_0)$$

Поч. услови: $E_K(\varphi_0) = 0$, $E_p(\varphi_0) = mg\frac{\ell}{2}\cdot\sin\varphi_0 = \frac{mg\ell}{2}\underbrace{\sin 30^\circ}_{1/2} = \frac{1}{4}mg\ell$

$$\frac{1}{6}m\ell^2\dot\varphi^2 + \frac{1}{2}mg\ell\sin\varphi = 0 + \frac{1}{4}mg\ell \;/:\left(\frac{1}{6}m\ell^2\right)$$

$$\dot\varphi^2 + 3\frac{g}{\ell}\sin\varphi = \frac{3}{2}\frac{g}{\ell} \quad \boxed{\dot\varphi(\varphi) = \sqrt{3\frac{g}{\ell}\left(\underbrace{\frac{1}{2}}_{\sin\varphi_0} - \sin\varphi\right)}}$$

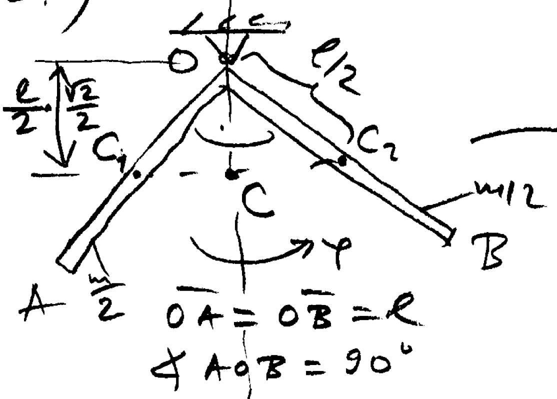
При $\varphi = 0$, $\dot\varphi(\varphi=0) = \sqrt{\frac{3}{2}\frac{g}{\ell}}$

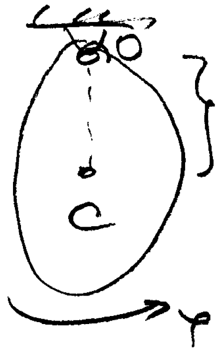
$$v_A(\varphi=0) = \dot y_A(\varphi=0) \;, \quad y_A = \ell\cdot\sin\varphi \;, \quad \dot y_A = \ell\cos\varphi\cdot\dot\varphi \quad \dot y_A(\varphi=0) = \ell\cdot\underbrace{\cos 0}_{1}\cdot\dot\varphi(0)$$

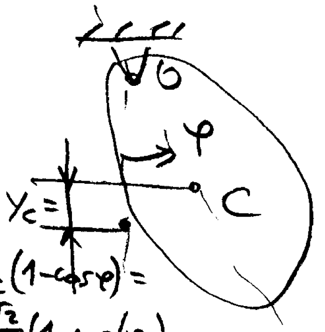
$$\boxed{v_A(\varphi=0) = \ell\cdot\sqrt{\frac{3}{2}\frac{g}{\ell}} = \sqrt{\frac{3}{2}g\ell}}$$ – брзина на A пред ударот

3.)

еквивалентно физичко клатно







$$y_c = l_{oc}(1-\cos\varphi) = \frac{\ell\sqrt{2}}{4}(1-\cos\varphi)$$

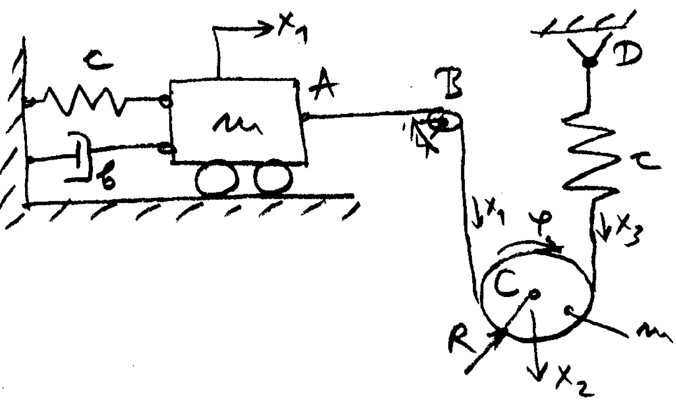
$$E_K = \frac{1}{2} J_0 \cdot \dot{\varphi}^2; \quad J_0 = \frac{1}{3}\cdot\frac{m}{2}\cdot\ell^2 + \frac{1}{3}\cdot\frac{m}{2}\ell^2 = \frac{1}{3} m\ell^2$$

$$\boxed{E_K = \frac{1}{6} m\ell^2\dot{\varphi}^2} \qquad E_p = mg\,y_c = mg\ell\frac{\sqrt{2}}{4}\underbrace{(1-\cos\varphi)}_{\approx \frac{\varphi^2}{2}} \approx \frac{\sqrt{2}}{8} mg\ell\varphi^2$$

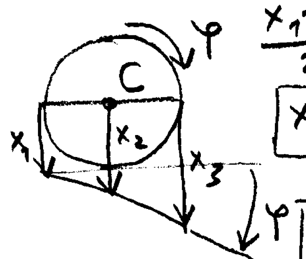
$$\frac{d}{dt}\frac{\partial E_K}{\partial\dot{\varphi}} - \cancel{\frac{\partial E_K}{\partial\varphi}}_{0} + \frac{\partial E_p}{\partial\varphi} = 0 \qquad \frac{1}{3}m\ell^2\ddot{\varphi} + \frac{\sqrt{2}}{4}mg\ell\varphi = 0 \quad /:\left(\frac{1}{3}m\ell^2\right)$$

$$\ddot{\varphi} + \underbrace{\frac{3\sqrt{2}}{4}\frac{g}{\ell}}_{\omega^2}\cdot\varphi = 0 \qquad \ddot{\varphi} + \omega^2\varphi = 0 \qquad \boxed{\omega = \sqrt{\frac{3\sqrt{2}}{4}\frac{g}{\ell}}}$$

4.)



2 сл. на сл., x_1, x_2 – ген. коорд.
x_3, φ – зависни промењиви



$$\frac{x_1 + x_3}{2} = x_2 \rightarrow x_3$$

$$\boxed{x_3 = 2x_2 - x_1}$$

$$x_1 + R\varphi = x_2 \rightarrow$$

$$\boxed{\varphi = \frac{x_2 - x_1}{R}}$$

$$E_K = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{2}J_c\cdot\dot{\varphi}^2 = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{2}\cdot\underbrace{\frac{1}{2}m\cancel{R^2}}_{J_c}\cdot\underbrace{\left(\frac{\dot{x}_2-\dot{x}_1}{\cancel{R}}\right)^2}_{\dot{\varphi}^2}$$

$$E_K = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{4}m\left(\dot{x}_2^2 - 2\dot{x}_1\dot{x}_2 + \dot{x}_1^2\right)$$

$$\boxed{E_K = \frac{3}{4}m\dot{x}_1^2 - \frac{1}{2}m\dot{x}_1\dot{x}_2 + \frac{3}{4}m\dot{x}_2^2} \; ; \; \boxed{\Phi = \frac{1}{2}b\cdot\dot{x}_1^2}$$

$$E_p = \frac{1}{2}c\cdot(x_1 + \Delta l_{1st})^2 - \frac{1}{2}c\Delta l_{1st}^2 + \frac{1}{2}c(x_3 + \Delta l_{2st})^2 - \frac{1}{2}c\Delta l_{2st}^2 - mg x_2$$

$$E_p = \frac{1}{2}cx_1^2 + c\Delta l_{1st}x_1 + \cancel{\frac{1}{2}c\Delta l_{1st}^2} - \cancel{\frac{1}{2}c\Delta l_{1st}^2} + \frac{1}{2}cx_3^2 + c\Delta l_{2st}x_3 + \cancel{\frac{1}{2}c\Delta l_{2st}^2} - \cancel{\frac{1}{2}c\Delta l_{2st}^2} - mgx_2$$

$$E_p = \frac{1}{2}cx_1^2 + c\Delta l_{1st}x_1 + \frac{1}{2}c(2x_2 - x_1)^2 + c\Delta l_{2st}(2x_2 - x_1) - mg x_2$$

$$E_p = \frac{1}{2}cx_1^2 + c\Delta l_{1st}\cdot x_1 + 2cx_2^2 - 2cx_1x_2 + \frac{1}{2}cx_1^2 + c\Delta l_{2st}(2x_2 - x_1) - mg\,x_2$$ (3)

$$E_p = cx_1^2 - 2cx_1x_2 + 2cx_2^2 + (\underbrace{c\Delta l_{1st} - c\Delta l_{2st}}_{0})x_1 + (\underbrace{2c\Delta l_{2st} - mg}_{0})\cdot x_2$$

$$\left.\frac{\partial E_p}{\partial x_1}\right|_{x_1,x_2=0} = 0 \;,\; \left.\frac{\partial E_p}{\partial x_2}\right|_{x_1,x_2=0} = 0$$

$$\left.\frac{\partial E_p}{\partial x_1}\right|_{\substack{x_1=0\\x_2=0}} = 2cx_1 - 2cx_2 + c\Delta l_{1st} - c\Delta l_{2st}\Big|_{\substack{x_1=0\\x_2=0}} = c\Delta l_{1st} - c\Delta l_{2st} = 0$$

$$\left.\frac{\partial E_p}{\partial x_2}\right|_{\substack{x_1=0\\x_2=0}} = -2cx_1 + 4cx_2 + 2c\Delta l_{2st} - mg\Big|_{\substack{x_1=0\\x_2=0}} = 2c\Delta l_{2st} - mg = 0$$

$$\boxed{E_p = cx_1^2 - 2cx_1x_2 + 2cx_2^2}$$

$$\begin{cases} \dfrac{d}{dt}\dfrac{\partial E_k}{\partial \dot{x}_1} + \dfrac{\partial \phi}{\partial \dot{x}_1} + \dfrac{\partial E_p}{\partial x_1} = 0 \\ \dfrac{d}{dt}\dfrac{\partial E_k}{\partial \dot{x}_2} + \dfrac{\partial \phi}{\partial \dot{x}_2} + \dfrac{\partial E_p}{\partial x_2} = 0 \end{cases}$$

$$\begin{cases} \frac{3}{2}m\ddot{x}_1 - \frac{1}{2}m\ddot{x}_2 + b\dot{x}_1 + 2cx_1 - 2cx_2 = 0 \\ -\frac{1}{2}m\ddot{x}_1 + \frac{3}{2}m\ddot{x}_2 - 2cx_1 + 4cx_2 = 0 \end{cases}$$

$$\begin{cases} x_1(t) = A\cdot e^{\lambda t} \\ x_2(t) = B\cdot e^{\lambda t} \end{cases} \quad \begin{cases} \dot{x}_1 = A\lambda\cdot e^{\lambda t} \\ \dot{x}_2 = B\lambda\cdot e^{\lambda t} \end{cases} \quad \begin{cases} \ddot{x}_1 = A\lambda^2 e^{\lambda t} \\ \ddot{x}_2 = B\lambda^2\cdot e^{\lambda t} \end{cases}$$

$$\begin{cases} \left[\left(\frac{3}{2}m\lambda^2 + b\lambda + 2c\right)A + \left(-\frac{1}{2}m\lambda^2 - 2c\right)B\right]\cdot e^{\lambda t} = 0 \\ \underbrace{\left[\left(-\frac{1}{2}m\lambda^2 - 2c\right)A + \left(\frac{3}{2}m\lambda^2 + 4c\right)B\right]}_{=0}\cdot \underbrace{e^{\lambda t}}_{\neq 0} = 0 \end{cases}$$

$$\Delta(\lambda) = \begin{vmatrix} \frac{3}{2}m\lambda^2 + b\lambda + 2c & -\left(\frac{1}{2}m\lambda^2 + 2c\right) \\ -\left(\frac{1}{2}m\lambda^2 + 2c\right) & \frac{3}{2}m\lambda^2 + 4c \end{vmatrix} = 0$$

$$\Delta(\lambda) = \left(\frac{3}{2}m\lambda^2 + b\lambda + 2c\right)\cdot\left(\frac{3}{2}m\lambda^2 + 4c\right) - \left(\frac{1}{2}m\lambda^2 + 2c\right)^2 = 0 \rightarrow \lambda_{1,2}$$