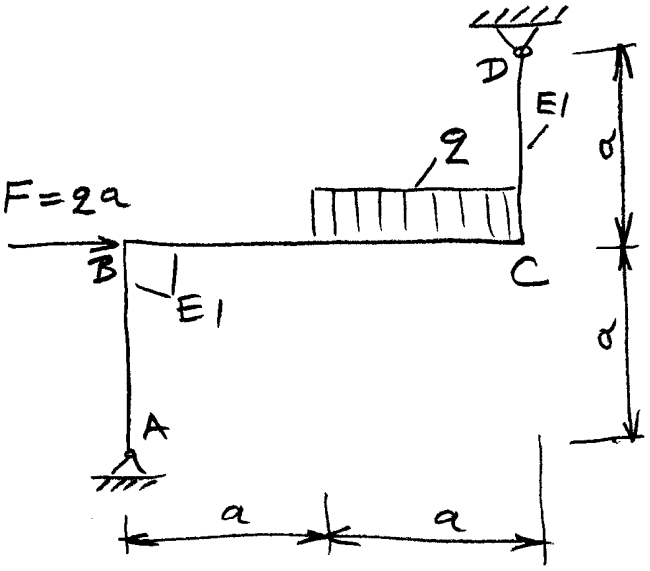
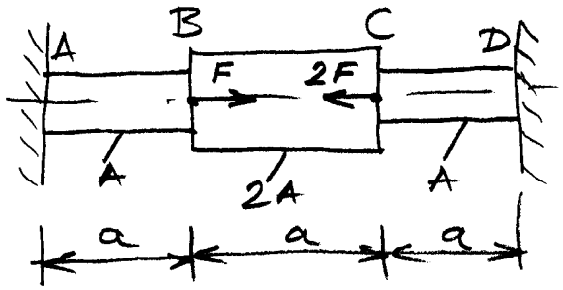
ЈАКОСТ НА КОНСТРУКЦИИТЕ

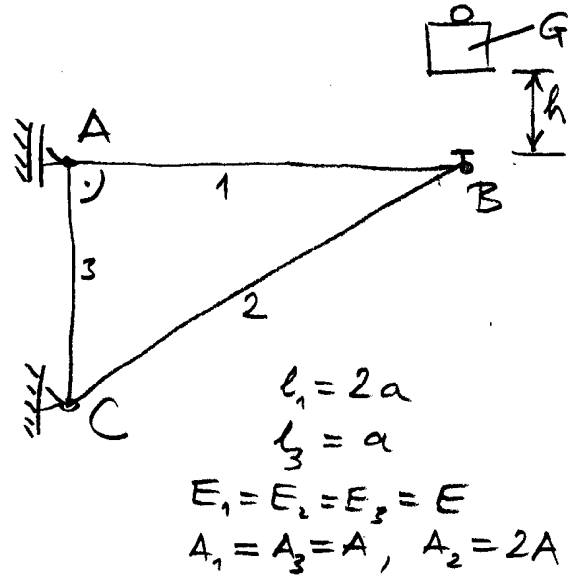


1.) За дадениот носач да се конструираат дијаграмите на нападните моменти на свиткување M, попречните сили Q и надолжните сили N. (вкупно 50 б., од тоа 30 б. за наоѓање на одвишната реакција и 20 б. за конструкција на дијаграмите на пресечните големини).



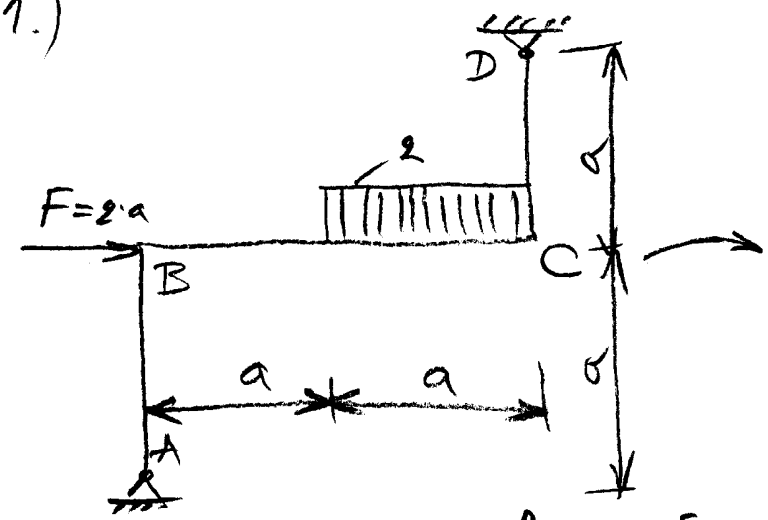
2.) Сложениот стап ABCD, со променлив попречен пресек, е вклештен на своите краеви и товарен со спротивно насочените сили F и 2F во точките B и C. Модулот на еластичност на материјалот на стапот е E. Потребно е да се најдат:

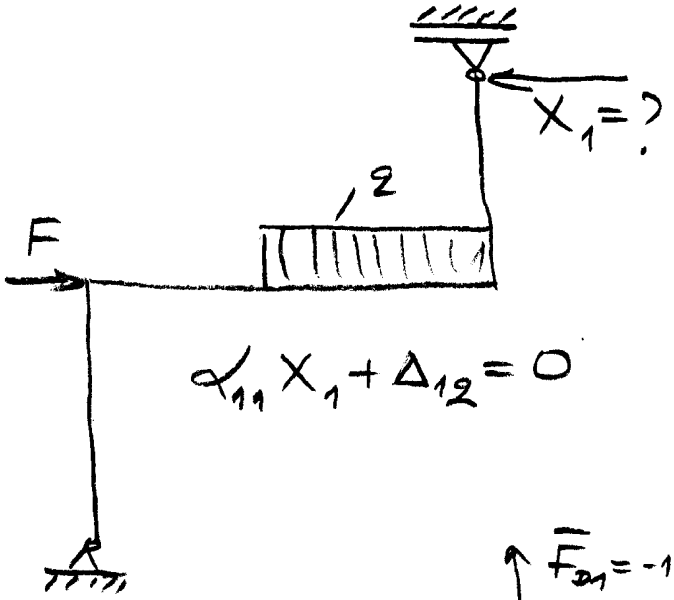
- реакциите во потпорите A и D. (10 б.)
- дијаграмот на надолжните сили N (10 б.)
- насоката и големината на хоризонталното поместување на точката B. (10 б.)



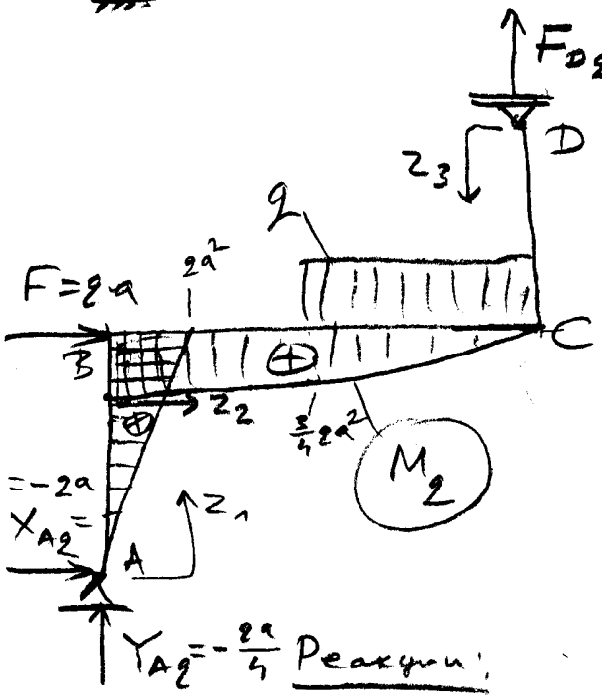
3.) Да се најдат динамичкиот коефициент k_d (15 б.) и динамичката сила (5 б.) во стапот 2 од дадениот решеткаст носач, при удар на товар со тежина G кој слободно паѓа од висина h.

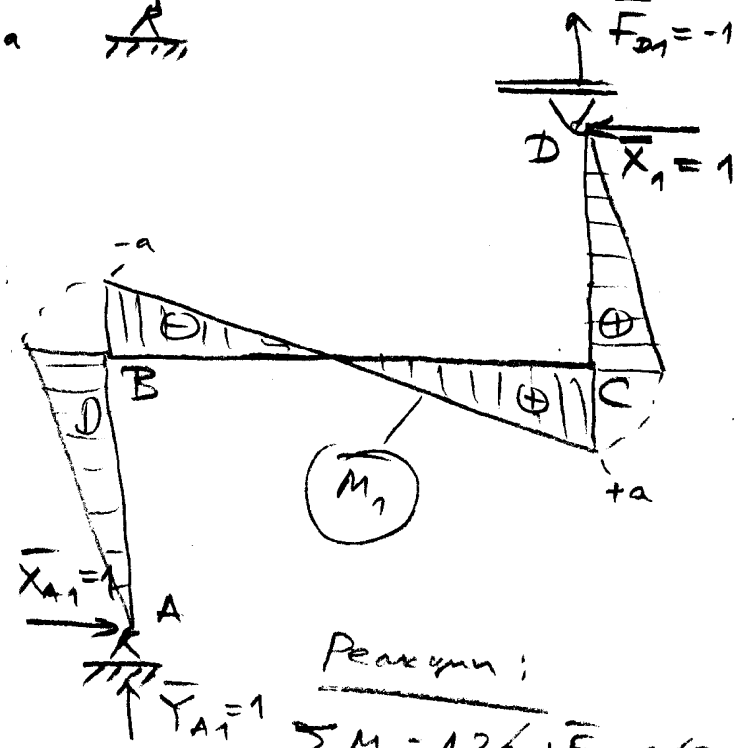
1.)





$$\delta_{11} X_1 + \Delta_{1q} = 0$$





Реакции:

$$\sum M_A = -F \cdot a - q \cdot a \cdot \frac{3a}{2} + F_{Dq} \cdot 2a = 0$$

$$F_{Dq} = \frac{1}{2a}\left(q \cdot a^2 + \frac{3}{2} q a^2\right) = \frac{5}{4} q \cdot a$$

$$Y_A = Y_{Aq} - q \cdot a + F_{Dq} = 0$$

$$Y_{Aq} = q \cdot a - \frac{5}{4} q a = -\frac{1}{4} q a$$

$$\sum X = F + X_{Aq} = qa + X_{Aq} = 0$$

$$X_{Aq} = -2a$$

Реакции:

$$\sum \bar{M}_A = 1 \cdot 2 + \bar{F}_{D1} \cdot 2 = 0$$

$$\bar{F}_{D1} = -1$$

$$\sum Y = \bar{Y}_{A1} + \bar{F}_{D1} = \bar{Y}_{A1} - 1 = 0$$

$$\bar{Y}_{A1} = 1$$

$$\sum X = \bar{X}_{A1} - \bar{X}_1 = \bar{X}_{A1} - 1 = 0$$

$$\bar{X}_{A1} = 1$$

Нат. моменти

$$M_q = \begin{cases} q a \cdot z_1, & 0 \le z_1 \le a \\ -\frac{qa}{4} \cdot z_2 + qa^2, & 0 \le z_2 \le a \\ -\frac{qa}{4} z_2 + qa^2 - \frac{1}{2} q (z_2 - a)^2, & a \le z_2 \le 2a \\ 0, & 0 \le z_3 \le a \end{cases}$$

$$\bar{M}_1 = \begin{cases} -z_1, & 0 \le z_1 \le a \\ z_2 - a, & 0 \le z_2 \le a \\ z_2 - a, & a \le z_2 \le 2a \\ z_3, & 0 \le z_3 \le a \end{cases}$$

Вычисление коэф.

$$\alpha_{11} = \sum \int \frac{\bar{M}_1^2}{EI} dz = \frac{1}{EI}\left[\int_0^a z_1^2\, dz_1 + \int_0^{2a} (z_2-a)^2 dz_2 + \int_0^a z_3^2\, dz_3\right] =$$

$$= \frac{1}{EI}\left[\frac{z_1^3}{3}\Big|_0^a + \frac{(z_2-a)^3}{3}\Big|_0^{2a} + \frac{z_3^3}{3}\Big|_0^a\right] = \frac{1}{EI}\left[\frac{a^3}{3} + \frac{a^3-(-a^3)}{3} + \frac{a^3}{3}\right]$$

$$\alpha_{11} = \frac{4}{3}\frac{a^3}{EI}$$

$$\Delta_{1q} = \sum \int \frac{M_q \cdot \bar{M}_1}{EI} dz = \frac{1}{EI}\left[-\frac{qa}{2}\int_0^a z_1^2\, dz_1 + \frac{qa}{2}\int_0^a \left(a-\frac{z_2}{4}\right)\cdot(z_2-a)\, dz_2 + \right.$$

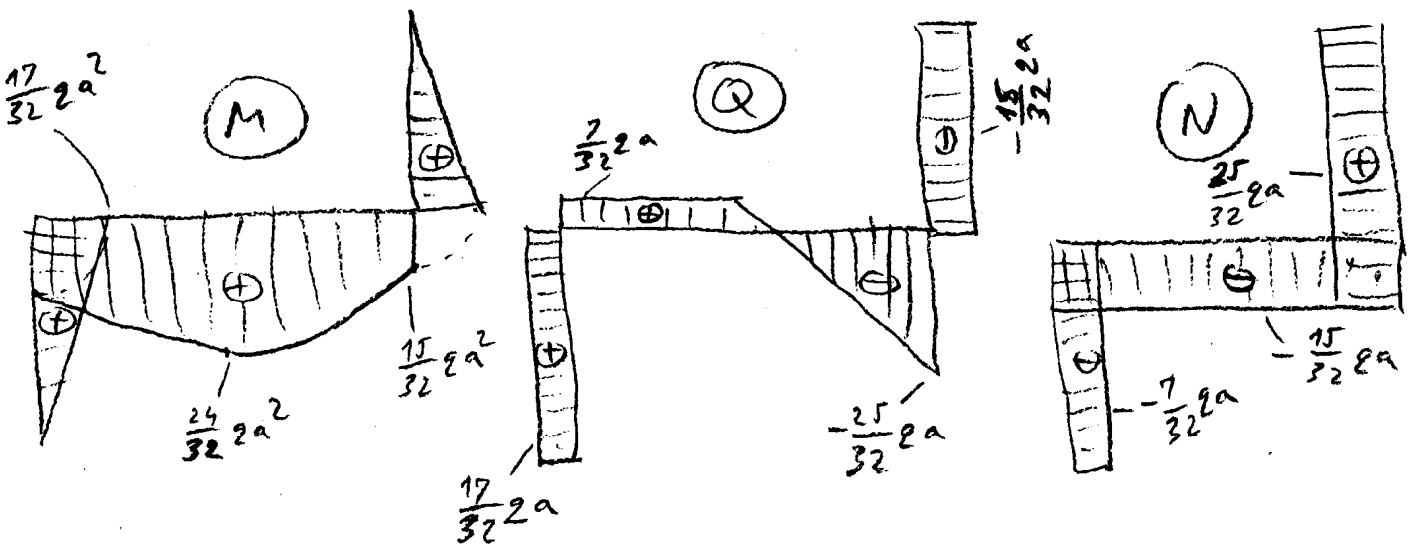
$$\left. + \frac{qa}{2}\int_a^{2a}\left(-\frac{z_2}{4}+a-\frac{a}{2}\left(\frac{z_2}{a}-1\right)^2\right)\cdot(z_2-a)\cdot dz_2\right] = \frac{1}{EI}\left[-\frac{qa}{2}\underbrace{\frac{z_1^3}{3}\Big|_0^a}_{a^3/3} + \frac{qa^2}{2}\underbrace{\int_0^a z_2\cdot dz_2}_{a^2/2} - \right.$$

$$- \frac{qa}{8}\underbrace{\int_0^a z_2^2\, dz_2}_{a^3/3} - \frac{qa^2}{2}\underbrace{\int_0^a dz_2}_{a} + \frac{qa^2}{8}\underbrace{\int_0^a z_2\cdot dz_2}_{a^2/2} - \frac{qa}{8}\underbrace{\int_a^{2a} z^2\, dz_2}_{\frac{7}{3}a^3} + \frac{qa^2}{2}\underbrace{\int_a^{2a} z_2\cdot dz_2}_{\frac{3}{2}a^2} +$$

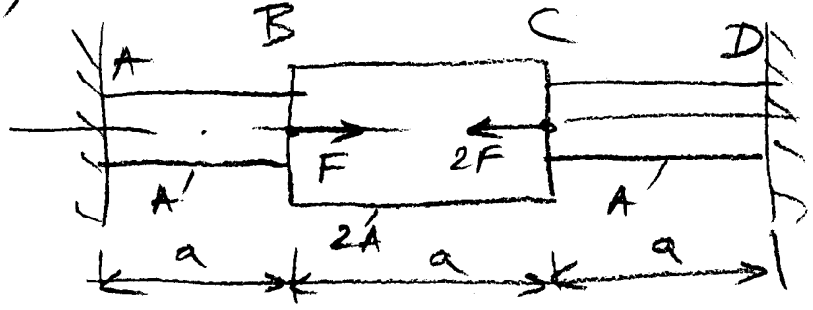
$$\left. + \frac{qa^2}{8}\underbrace{\int_a^{2a} z_2\cdot dz_2}_{\frac{3}{2}a^2} - \frac{qa^2}{2}\underbrace{\int_a^{2a} dz_2}_{a} + \frac{q}{2}\underbrace{\int_a^{2a}(z_2-a)^3\cdot dz_2}_{\frac{qa^4}{8}}\right] = \frac{qa^4}{EI}\times$$

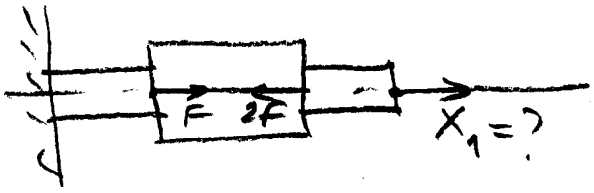
$$\times\left(-\frac{1}{3}+\frac{1}{2}-\frac{1}{12}-1+\frac{1}{8}-\frac{7}{12}+\frac{3}{2}+\frac{3}{8}-1-\frac{1}{8}\right) = -\frac{15}{24}\cdot\frac{qa^4}{EI}$$

$$\alpha_{11}X_1 + \Delta_{1q} = 0 \qquad \frac{4}{3}\frac{a^3}{EI}\cdot X_1 - \frac{15}{24}\frac{qa^4}{EI} = 0 \qquad \boxed{X_1 = \frac{15}{32}qa}$$

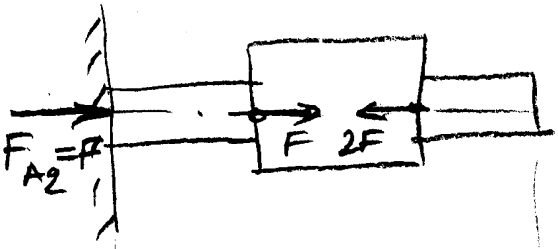


2.)



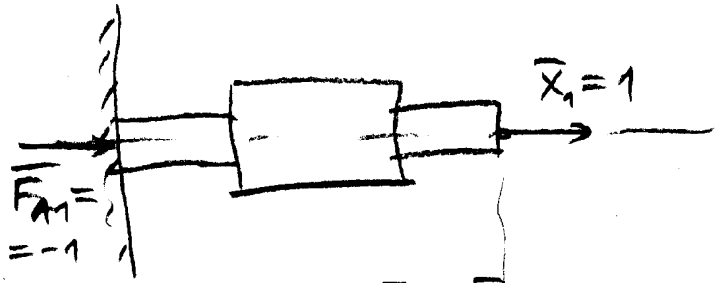


$$\alpha_{11} X_1 + \Delta_{12} = 0$$



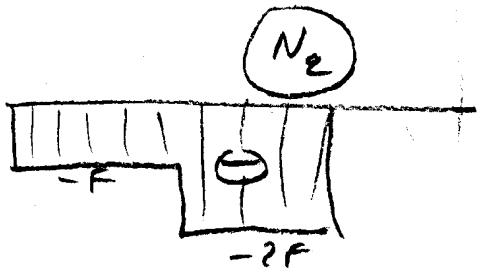
$$\Sigma X = F_{A2} + F - 2F = 0$$

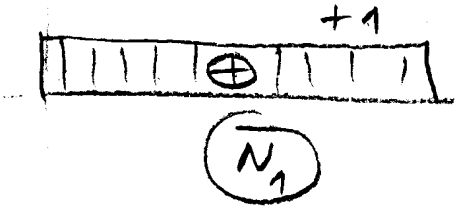
$$F_{A2} = F$$



$$\Sigma X = \bar{F}_{A1} + \bar{X}_1 = \bar{F}_{A1} + 1 = 0$$

$$\bar{F}_{A1} = -1$$





a.) $$\alpha_{11} = \sum \frac{\bar{N}_1^2 \cdot l_i}{E_i A_i} = \frac{1}{EA}\left(1^2 \cdot a + \frac{1}{2} \cdot 1^2 \cdot a + 1^2 \cdot a\right) = \frac{5}{2}\frac{a}{EA}$$

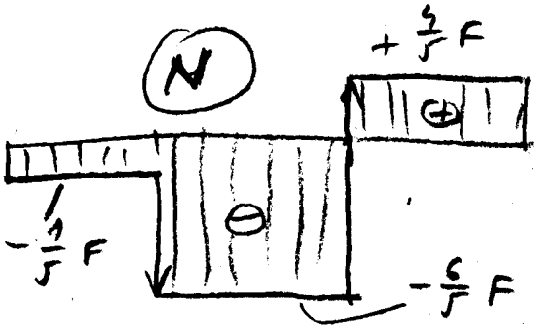
$$\Delta_{12} = \sum \frac{N_2 \cdot \bar{N}_1 l_i}{E_i A_i} = \frac{1}{EA}\left(-F \cdot a - \frac{2F \cdot 1 \cdot a}{2} - 0\right) = \frac{-2Fa}{EA}$$

$$\alpha_{11} X_1 + \Delta_{12} = 0 \qquad \frac{5}{2}\frac{a}{EA} X_1 - \frac{2Fa}{EA} = 0 \qquad \boxed{X_1 = \frac{4}{5} F}$$

$$F_A = F_{A2} + X_1 \cdot \bar{F}_{A1} = F + (-1) \cdot \frac{4}{5} F = \frac{1}{5} F$$

$$F_D = 0 + X_1 = \frac{4}{5} F \qquad ; \qquad N = N_2 + X_1 \cdot \bar{N}_1$$

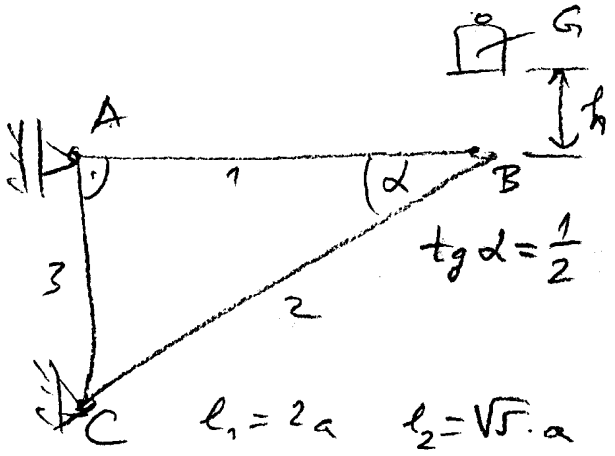
δ)



l.) $$\delta_B = \frac{N_{A-B} \cdot a}{EA} = \frac{-\frac{1}{5} F \cdot a}{EA} = -\frac{Fa}{5EA}$$

(наляво)

3.)



$$tg\,\alpha = \frac{1}{2}$$

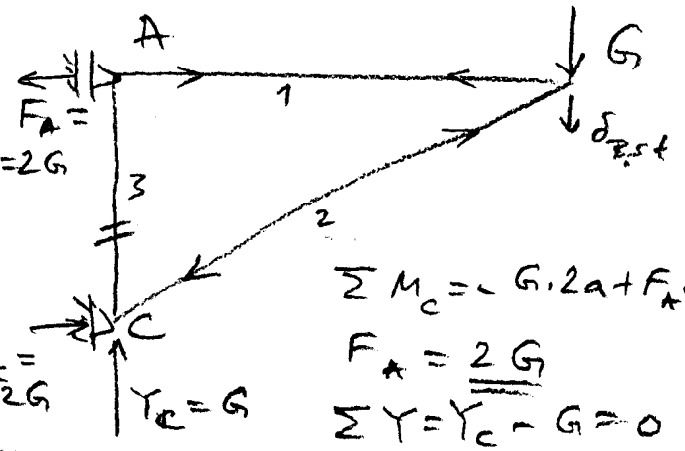
$$l_1 = 2a \qquad l_2 = \sqrt{5}\cdot a$$

$$l_3 = a$$

$$E_1 = E_2 = E_3 = E$$

$$A_1 = A_3 = A\,, \quad A_2 = 2A$$

$$k_d = 1 + \sqrt{1 + \frac{2h}{\delta_{3.st}}}$$



$$\Sigma M_C = -G\cdot 2a + F_A\cdot a = 0$$

$$F_A = \underline{2G}$$

$$\Sigma Y = Y_C - G = 0$$

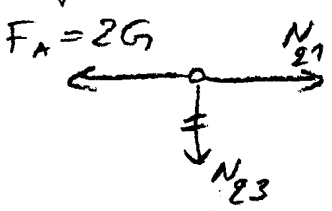
$$Y_C = \underline{G}$$

$$\Sigma X = -F_A + X_C = 0$$

$$X_C = \underline{2G}$$

Сили во сите шипки:

Јазол "A"

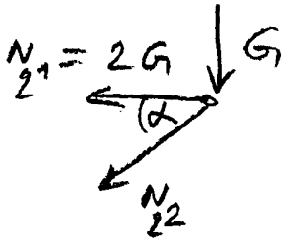


$$\Sigma Y = -N_{23} = 0$$

$$\Sigma X = -2G + N_{21} = 0$$

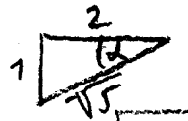
$$\boxed{N_{21} = 2G}$$

Јазол "B"



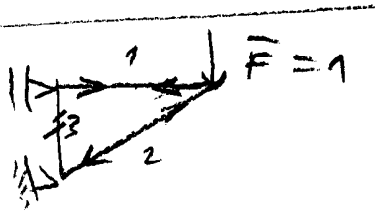
$$\Sigma Y = -G - N_{22}\cdot \sin\alpha = 0$$

$$N_{22} = -\frac{G}{\sin\alpha}$$



$$\sin\alpha = \frac{1}{\sqrt{5}}$$

$$\boxed{N_{22} = -\frac{\sqrt{5}}{5}\cdot G}$$



$$\bar{N}_1 = 2$$

$$\bar{N}_2 = -\frac{\sqrt{5}}{5}$$

$$\bar{N}_3 = 0$$

$$\delta_{B.st} = \sum \frac{N_{2i}\bar{N}_i\cdot l_i}{E_i A_i} = \frac{2G\cdot 2\cdot 2a}{EA} + \frac{-\frac{\sqrt{5}}{5}G\cdot\left(-\frac{\sqrt{5}}{5}\right)\cdot\sqrt{5}\cdot a}{2EA} + \frac{0\cdot 0\cdot a}{E\cdot A}$$

$$\delta_{B.st} = \left(8 + \frac{\sqrt{5}}{10}\right)\frac{G\cdot a}{EA}$$

$$\boxed{k_d = 1 + \sqrt{1 + \frac{2h}{\left(8 + \frac{\sqrt{5}}{10}\right)\frac{Ga}{EA}}}}$$

$$N_{22.dn} = k_d\cdot N_{22.st} = k_d\cdot\left(-\frac{\sqrt{5}}{5}G\right) = \left[1 + \sqrt{1 + \frac{2h}{\left(8+\frac{\sqrt{5}}{10}\right)\frac{Ga}{EA}}}\right]\cdot\left(-\frac{\sqrt{5}}{5}G\right)$$