CONTRIBUTION TO NEW APPROACHES ON MODELLING OF URBAN ARTERIALS WITH ADVANCED CELL TRANSMISSION MODEL (CTM)
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Traffic operations of particular nodes-intersections that comprise an urban arterial reflects, in a great mass, the performance measure of the overall traffic network of a city and beyond. As a major part of the arterials, intersections in a huge mass are considered as the main cause of the regular flow or the interruptions of traffic of the arterials. Modelling and simulations of any particular segment of the road or of any road network have always played a key role, in identification of the problems such as oversaturation of traffic flow, which phenomenon is known as the state where the vehicle groups follow a trip with high density rates, low speed and an unsatisfactory level of comfort caused by long waiting times. As to the congestion, in worst cases it can be recognized as the phenomena of building of any long queue which extends between two neighboring intersections, and blocks the traffic flow for any certain time.

During the history of seeking various analytical traffic engineering techniques, lots of traffic simulation tools have been proven to have potential solutions in traffic problems. Related to the controlling form of intersections, whatever they were, un-signalized or signalized, adequate techniques were adopt for analysis and identifications of various problems. These simulation techniques have evolved with traffic flow models that describe traffic flow by different aspects, and as the most popular classification of them is the microscopic and macroscopic traffic flow.

In microscopic models the process of traffic flow is described on the level of the individual entities or driver units (vehicles) and the interactions between these units explicitly modeled. The traffic flow process is the collective behavior of all the units together while macroscopic models consider the traffic as the continuum as fluid flow by the characteristic quantities such as characteristics as, speed, traffic density and traffic flow or volume at point and time respectively. The earliest macroscopic model was proposed by Lighthill and Whitham (1955); Richards in (1956) referred as LWR model.

Some other forms of traffic flow modeling are discretized models when the main traffic variables of LWR model are discretized in time length dimensions. In the frame of the discretized models one and very extensively used on the two past decades is the cell transmission model (CTM).

This dissertations purpose is the development of an enhanced CTM model that will be able to model traffic conditions particularly on the urban arterials, in both free and saturated flow state. CTM model was first developed by Daganzo (1994), but since that time due to the fact of its simplicity of expression of traffic parameters, has become a popular model to researchers of the modelling disciplines. Author has found the new way to overcome the difficulties of partial differentiating by partitioning the road segments and adopting the fundamental diagram of flow and density. So, models that derived from the first general CTM hopefully have served as robust tools for addressing traffic problems although their perfections is not achieved yet.

This PhD dissertation is comprised of 9 chapters, 212 pages, 82 figures and 12 tables. There are listed around 65 references regarding the literature review. For the aim of concretization 10 appendices are provides also. During the preparation of this dissertation many research papers are achieved to be published and presented in international scientific journals and conferences. The list of published papers is following:


1. **SUBJECT, OBJECTIVES, HYPOTHESIS, USED METHODOLOGY, STRUCTURE AND EXPECTED RESULTS**

**Subject** of this doctoral dissertation is the improvement of CTM (Cell Transmission Model) based macroscopic model through which the estimation and update of traffic parameters as density, flow in time and distance dimension is enabled.

**The main purpose** of building of this model is its application not only on the simple composition such as highways but to enable it to be used to complex composition as diverges and diverges of intersections. The specific objectives of this dissertation are:

- a. Analysis of FDR diagrams that describes the relationship of fundamental parameters of traffic flow for each part of urban arterial,
- b. Inclusion of these FDR diagrams on the proposed CTM model,
- c. Capability to model the traffic interruptions (as result of red interval) and different blockages,
- d. Capability to model the reduced discharge flow rates at the beginning of green interval (as results as startup lost times at beginning of green interval),
- e. Validation of density and flow of each cell on every time step,
- f. Integration of a filter on it, for traffic parameter prediction that are necessary on real time traffic control.

**Hypothesis** of research. Based on the research objectives, the below hypothesis were tested and proven:

- **Hypothesis Ho:** The difference between delays of original and new CTM model is equal to zero, that means the models are same.
- **Hypothesis H₁:** the difference between delays of both original and new CTM model is different from zero.
- **Hypothesis Ho:** The samples of Inter-cell flows and flows of middle cells come from the same population.
- **Hypothesis H₁:** The samples do not come from the same population
- **Hypothesis Ho:** The samples of Inter-cell flows and flows of cells upstream to intersection come from the same population.
- **Hypothesis H₁:** The samples do not come from the same population

**Survey Methods and Techniques:** through the work of this doctoral dissertation, are applied qualitative and quantitative methods: inductive and deductive analysis and synthesis methods, in order to achieve the target results. Application of a smart application for data processing and two analysis software is done throughout the work of this dissertation. The research provides three main parts:

- Filed data collection of traffic flow through video recording and surveys with the aim of building of fundamental diagrams for the most representative samples of the subjected segment. Excel processing of data and obtaining of diagrams,
**New model algorithm creation and its code in a programming environment-CTM calculator building.**

**Testing of initial conditions, free flow conditions and congested conditions. Analysis of the possibilities to integrate a filter (Kalman Filter) on the model for prediction purposes.**

**Research Results:** Since the essential purpose of the research is the improvement of the CTM based model, toward ability of description of meanwhile oscillated traffic flow on different parts of road segments, the expected results are:

- The observation of changeable traffic parameters on both nodes and segments and inclusions of new definitions on the improved model
- The model of the principles and legality as approximate as possible,
- Validation of the model as base for traffic parameter prediction, for application in real time traffic control and different strategies for solutions of congestion problems in urban networks.

### 2. LITERATURE REVIEW

In the second chapter a related literature review and the achievement done so far regarding the CTM based model is done. The nowadays Intelligent Transportation Systems (ITS) require on line information of traffic parameters and during the development of the CTM models many benefits are achieved from them toward this aspect. In this chapter is given a brief review by intentionally ordering, firstly the researches related to highways model and then the arterials, because the first ones have date immediately after the first version of CTM, following by researches with regard to complex nodes-intersections of urban arterials.

### 3. CTM MODELS OF URBAN ARTERIALS

In third chapter are given the main principles of CTM model. According to first part of CTM, the most important definition is the inter cell flow, or flow between cells (i.e. $q_{i-1 \rightarrow i}(t)$ or $q_{i \rightarrow i+1}(t)$ of the example of fig. 1).

![Diagram of three cells with inter-cell flow](source)

**Fig. 1. Simple composition of three cells, inter-cell flow between $q_{i-1 \rightarrow i}$ and $q_{i \rightarrow i+1}$**

Source: Original from Author

A cell can maximally receive a number of vehicles, which their adding should not exceed the maximal number of vehicles that can be present on it during time $t$, or a number of vehicles equal to the capacity flow or the a number of vehicles that the empty space of cell can accept during time $t$.

$$q_{i-1 \rightarrow i}(t) = \min\{n_{i-1}(t), Q_i(t), \delta N_i(k) - n_i(k)\}$$

Where:

- $n_{i-1}(t)$, is the number of vehicles in cell $i-1$ at time $t$,
- $Q_i(t)$, is the capacity flow into $i$ for time interval $t$,
- $N_i(t)$, is the maximal number of vehicles that can be present on cell $i$ during time $t$,

~6~
\( N_i(t) - n_i(t) \) is the amount of empty space in cell \( i \) at time \( t \)

\( \delta = \frac{w}{v} \), is the ratio between wave propagation speed and free flow velocity, and \( \delta = 1 \), if \( n_{i-1}(t) \leq Q_i(t) \) and \( \delta \neq 1 \), if \( n_{i-1}(t) \geq Q_i(t) \).

With the elaboration of the value of \( \delta \), we can directly write the equation in another form.

### 4. CHARACTERISTICS PARAMETERS OF CTM FOR INTERSECTION AND SEGMENT CONFIGURATION

Unlike original CTM when these characteristic parameters were constant at all discretized parts and during all time steps, here we face with the need of a calibration framework, from which will obtained the main parameters depending on the dynamic of traffic conditions belonging on the parts of arterial or in any particular cell of it as well as for different evolution times, i.e. red/green interval, beginning of green, end of green, etc. Following are given parameters incorporated in the model. In chapter three through Table 1 are presented the parameters that must be included on the proposed model.

<table>
<thead>
<tr>
<th>Table 1. Consideration of Changeable Parameters/ Calibration</th>
<th>Original CTM</th>
<th>New CTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complexity of road configuration</td>
<td>No</td>
<td>Yes Merge/diverge</td>
</tr>
<tr>
<td>Inter Cell Flow</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Flow Capacity</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Storage Capacity</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Free Flow Speed</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Backward Speed</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Critical Density</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Queuing Discharge Headways</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Shock Wave Speeds</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Source: Original from Author
Prior to methodology development for proposed CTM model, different theories of new definitions are reviewed:

- New sending and receiving functions (Sending and Receiving Functions) for cells upstream to intersection,
- Reduced flow rates of permitted left turn lanes that are simultaneous with opposite through movements,

Related to the above description are given some schemes of the most common cases of the operation of traffic signals. Following are given the main steps of the model algorithm.
Steps of the CTM Algorithm
Source: Original from Author
5. DATA COLLECTION METHODOLOGY FOR OBTAINING DIAGRAM OF RELATIONSHIP BETWEEN FUNDAMENTAL PARAMETERS-FDR

In the fourth chapter a case study which provides the data collection methodology and development of the model in an urban segment is given. Case study is designed to evaluate the performance of original CTM and new proposed CTM model in terms of predicting the traffic flows, densities under some different scenarios as presented in the section below. A test bed- urban arterial is chosen to model the proposed CTM on it. Model is compiled within C# programming environment algorithm. Test bed of this dissertation is focused on the urban road network of the city of Prishtina, capitol of Kosovo. It is the urban segment of bulevard “Bill Clinton” an extension of the Highway M9- “Peja League”, along with three signalized intersection, one on ramp and one facility for both entry and exit operations. The map in three formats is depicted in fig.2. The arterial is of length 970 meters, with lane widths 3.2, meters on the first and second intersections and 3.0 meters on the third intersection.

![Fig.2. Urban Segment “Bill Clinton”/M9, Source: Geoportal Kosovo Map](image)

![Fig.3. CTM configuration of the Urban Segment Source: Original from Author](image)

5.1. Data Collection, FDR diagrams

Traffic data collection is done for every cell for purpose of compilation of fundamental diagram is realized through surveying with smart phone video recordings. The length of road cells are chosen of 25 meters long while. Initially, for the first vehicle in the queue at the beginning of green, is assigned the time between the start of green and the time when this vehicle (its front bumper) passed the stop line. For the other queued vehicles behind the first one, discharge headways is calculated...
as the elapsed time between two successive (one after other) passing the stop line. Through the times collected are obtained amount of flows, speeds and densities for every five second interval. Summarizing, for every single cell, are obtained 180 flow/densities data. (5 second intervals of 15 minute).

Calculation of travelled time and velocity for two vehicles with the aid of Stop Watch utilization is described and presented on Figure 4.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t1'1</td>
<td>1.27</td>
</tr>
<tr>
<td>t1'2</td>
<td>2.72</td>
</tr>
<tr>
<td>t2'1</td>
<td>1.27</td>
</tr>
<tr>
<td>t2'2</td>
<td>2.72</td>
</tr>
</tbody>
</table>

Fig. 4 Video recordings in “Bill Klinton” segment road and data processing with Stopwatch
Source: Original from Author

As can be seen from the table, for each relevant cell to the sample are obtained different values of the main traffic parameters, maximal flow, $Q_e$, critical density, $\rho_{cr}$, free flow speed, $V_f$ backward speed, $w$ all this because of different number of lanes (column two) which directly affects the value of the jam density, $\rho_J$, that on the case of three lanes has a value of $\rho_J=600$ [veh/km], while for two lanes $\rho_J=600=400$ [veh/km].
Table 2 FDR parameters for each sample/cell

<table>
<thead>
<tr>
<th>Sample</th>
<th>Relevant CELL</th>
<th>Nr.of Lanes</th>
<th>Qc [veh/hr]</th>
<th>ρc [veh/km]</th>
<th>Vf [km/hr]</th>
<th>w [km/hr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>(1.10-1.3) Middle</td>
<td>3</td>
<td>2880,00</td>
<td>147,00</td>
<td>20,00</td>
<td>6,35</td>
</tr>
<tr>
<td>[2]</td>
<td>(1.2&amp;1.1)</td>
<td>2</td>
<td>4230,00</td>
<td>115,00</td>
<td>37,00</td>
<td>14,84</td>
</tr>
<tr>
<td>[3]</td>
<td>(1.0) Merge</td>
<td>3</td>
<td>4320,00</td>
<td>117,00</td>
<td>37,00</td>
<td>8,94</td>
</tr>
<tr>
<td>[4]</td>
<td>(2.4) Middle</td>
<td>3</td>
<td>4320 (3600)</td>
<td>149 (113)</td>
<td>29 (32)</td>
<td>9,57(7,39)</td>
</tr>
<tr>
<td>[5]</td>
<td>(2.3)</td>
<td>3</td>
<td>3600,00</td>
<td>176,00</td>
<td>20,00</td>
<td>8,49</td>
</tr>
<tr>
<td>[6]</td>
<td>(2.2)</td>
<td>2</td>
<td>3600,00</td>
<td>164,00</td>
<td>22,00</td>
<td>15,25</td>
</tr>
<tr>
<td>[7]</td>
<td>(2.1)</td>
<td>2</td>
<td>2880,00</td>
<td>88,00</td>
<td>33,00</td>
<td>9,23</td>
</tr>
<tr>
<td>[8]</td>
<td>(2.0) Merge</td>
<td>2</td>
<td>3600,00</td>
<td>183,00</td>
<td>20,00</td>
<td>16,58</td>
</tr>
<tr>
<td>[9]</td>
<td>(3.2)</td>
<td>2</td>
<td>2880 (2160)</td>
<td>77 (102)</td>
<td>37 (21)</td>
<td>8,91(7,24)</td>
</tr>
<tr>
<td>[10]</td>
<td>(3.1)</td>
<td>2</td>
<td>2280,00</td>
<td>115,00</td>
<td>20,00</td>
<td>10,10</td>
</tr>
<tr>
<td>[11]</td>
<td>(3.7-3.6) Middle</td>
<td>3</td>
<td>2880,00</td>
<td>133,00</td>
<td>22,00</td>
<td>6,16</td>
</tr>
</tbody>
</table>

Source: Original from Author

6. RESULTS DISCUSSION AND EVALUATION OF MODEL PERFORMANCE BASED ON INITIAL CONDITIONS AND DIFFERENT TRAFFIC STATES

The purpose of the algorithm and setup is to obtain simulation results of the model for an overall time of fifteen minutes. Since we are obliged to fulfill the fundamental condition of the CTM model (a car with free flow speed is not allowed to traverse on the other cell during a time step), a time step of length 2.3 seconds is used. The actual cycle length of subjected intersections is 120 seconds, which means that one of them comprises of 52 time steps, the overall number of cycles within the simulation is 7.5 cycles, consequently the number of time steps in simulation run is 390. Our aim is not to perform a model simulation in separated cycles, but to obtain a continuance of the traffic state from cycle to cycle, nevertheless, the corresponding time steps to the main point as, beginning and ending of each green interval, had to be appointed on algorithm.

6.1. Initialization

Prior to experiments realization, an initialization as a foregoing process is performed in order to estimate the evolution of traffic through artery from the “zero point”. This “zero point” means the initial conditions of the whole artery, when all the cells are empty (density is zero), but the entry cell of the first segment (Cell 1.10) feeds the artery with average traffic flow until is get a stable state of traffic. A special survey is dedicated to the most representative cells i.e. initial cells on the segment (Cell 1.8, Cell 2.8 and Cell 3.8), cells upstream to intersections (Cell 1.1, Cell 2.1 and Cell 3.1) during the first cycle, consequently during first 52 time-steps. As it can be seen from the below diagrams, there is a faster increase of density (fill up with vehicles) of the cells of the first segment, that start immediately after the second time-step (fig.5), while there is an obvious delayed increasing
of density on the cells of the far away segments (fig.6 and fig.9). The earlier increasing of density is noted by after the tenth time-step to Cell 2.8, which reaches the average value in the middle of cycle (around time-step 25). The most delayed increasing of density is noted to the most far away cell (Cell 2.1) of the segment 2.

It is important to note that the high density values are never reached during the first cycle, by the last segment (segment3: Cell 3.8 to 3.1), that it is understood that a single cycle cannot be considered as a “feeding” or “warm up” period during which the artery sufficiently fills up. The Cell 3.1 positioned upstream to intersection 3, starts to fill up with vehicles by the end of cycle, at time-step 36 (fig.9).

An approximately equal evolution is obvious in the aspect of inter-cell flow too. Attention must be paid to the signal timing when it comes to the evaluation of the inter-cell flow.

The SBC of first intersection (Cell 1.1) starts to release vehicles from the first time-step (fig.6); the SBC of intersection 2 (Cell 2.1) starts to release vehicles by the time-step 23 when its interval green begins (fig.8) but no single vehicle is released from the Cell 3.1 during the first cycle.

The diagrams are accompanied with the tables at the end of this section that contains numerical values of the density and inter cell flow. As can be seen, a diagonal between positive and zero values is formed, that can be interpreted as a tendency of the increasing of the parameters for the distant cells, during later time-steps.

Fig. 5 Evolution of segment 1 during first cycle-Density
Source: New CTM Model

Fig. 6. Evolution of segment 1 during first cycle-Inter-Cell Flow
Source: New CTM Model

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6.2. Modelling of Light Traffic Conditions

Toward testing of suitability of new CTM to model different traffic conditions, special emphasis is given to the definition of entering flow or demand on the first cell, and the initial values of the density on each cell.
Light Traffic

Input values:

- Traffic demand: 0.3veh/sec,
- Signal timing plan as in fig.22
- Initial density 0.05veh/m,

In light traffic conditions, in great mass traffic flow is dictated by the traffic signal status, consequently we have a harmonious move of the density curve that follows the signal status. An increasing trend of density during green intervals and in opposite and a decreasing trend of density during green interval for each cell is noted. By the time-step 17 the initial cells (Cell 1.10, Cell 1.9, Cell 1.8 and Cell 1.7) of segment 1 reach the value of initial density (0.05veh/m) and maintain that value during the overall running time, (353 time-steps, equal; to seven cycles). The highest values of density are observed in the cells approaching the intersections, as a result of the influence of signal status. SBC of the first segment reach faster (at time-step 17) the density value 0.4veh/m that is equal to 10veh/SBC (3,3veh/lane), while the those of the next two segment reach the same value slowly (at time-step 33). A rapid increasing of density to overcoming of higher values is not observed during the whole simulation time during light traffic conditions.

6.3. Modelling of Medium to Congested Conditions

Input values:

- Traffic demand: 0.6veh/sec,
- Signal timing plan as in fig.22
- Initial density 0.1veh/m,

Explication of results (Evolution of Densities):

For creation of medium congestion of traffic flow, in this experiment is imposed a higher number of released vehicles to the first cell. Beside the higher traffic flow values, a double initial density is set to the CTM calculator. Unlike the light conditions, in medium congestion is obtained a rapid increasing of the density to each cell. Fluctuations of the density from the lowest to the highest values are obvious during the first three cycles to all cells of the artery. The lowest is zero and the highest takes values till 0.4veh/m, 0.5veh/m or 0.6veh/m, depending on the number of lanes that cell covers. After the first three cycles, density never get back to the lowest values but their fluctuations are between 0.4veh/m to 0.6veh/m.
Graphical results of Light Traffic Conditions

Fig. 11 Density of Cell 1.8
Source: New CTM Model

Fig. 12 Density of Cell 1.4
Source: New CTM Model

Fig. 13 Density of Cell 1.3
Source: New CTM Model

Fig. 14 Density of Cell 1.1- SBC
Source: New CTM Model
Fig. 15 Density of Cell 11L - Left Turn Lane  
Source: New CTM Model

Fig. 16 Density of Cell 2.8  
Source: New CTM Model

Fig. 17 Density of Cell 2.1 - SBC  
Source: New CTM Model

Fig. 18 Density of Cell 3.8  
Source: New CTM Model
Graphical Results of Congested Conditions
Fig. 22 Density of Cell 1.4
Source: New CTM Model

Fig. 23 Density of Cell 1.3
Source: New CTM Model

Fig. 24 Density of Cell 1.1-SBC
Source: New CTM Model

Fig. 25 Density of Cell 1.1L-Left Turn Lane
Source: New CTM Model
Fig. 26 Density of Cell 2.8
Source: New CTM Model

Fig. 27 Density of Cell 2.1-SBC
Source: New CTM Model

Fig. 28 Density of Cell 3.8
Source: New CTM Model

Fig. 29 Density of Cell 3.1L-Left Turn Lane
Source: New CTM Model
6.4. Hypothesis Testing

The delay difference between two models can be analyzed by formulating the problem through specification of null hypothesis. A statistical hypothesis is an assumption or a statement about one or two parameters of one or more populations. In this dissertation are done ten repetitions of runs in order to avoid the possibility of change of results from random chance. Table of delay results is given following.

<table>
<thead>
<tr>
<th></th>
<th>Orig. CTM</th>
<th>New CTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>D_{c(t)}</td>
<td>54.70</td>
<td>65.70</td>
</tr>
<tr>
<td></td>
<td>60.00</td>
<td>60.90</td>
</tr>
<tr>
<td></td>
<td>54.70</td>
<td>65.70</td>
</tr>
<tr>
<td></td>
<td>40.30</td>
<td>60.00</td>
</tr>
<tr>
<td></td>
<td>55.00</td>
<td>60.60</td>
</tr>
<tr>
<td></td>
<td>40.34</td>
<td>65.70</td>
</tr>
<tr>
<td></td>
<td>52.40</td>
<td>50.30</td>
</tr>
<tr>
<td></td>
<td>57.70</td>
<td>72.60</td>
</tr>
<tr>
<td></td>
<td>40.00</td>
<td>65.70</td>
</tr>
<tr>
<td></td>
<td>82.80</td>
<td>60.09</td>
</tr>
</tbody>
</table>

Finally can be concluded \( H_0 \) rejected, in favor of the preposition that difference between original CTM and CTM is statistically significant.

7. VELOCITY UPDATES BASED ON PROPOSED MODEL

7.1. Review and discussion on models of velocity-density relationship

Beside the possibility to update the densities of cells through time step, the CTM model offers the estimation of evolution of the velocities which may be an important factor of challenging real time prediction and control strategies. The Greenshields model is:

\[
v(\rho) = v_f \cdot \left[ 1 - \frac{\rho}{\rho_f} \right]
\]

Another model that describes a logarithmic relationship between velocity and density was proposed 60 years ago by Greenberg. It is based on analogy with hydrodynamics theory.

\[
v(\rho) = a \cdot \log \left[ \frac{\rho_f}{\rho} \right]
\]
A more or less modified Greenshields model was modeled from Pipes-Munjal and Drew in 1967 and in 1968, respectively. In conventional Greenshields model the ratio of density to jam density was empowered by coefficient and so could be derived a variety of models for velocity-density relationship.

\[
v(\rho) = v_f \cdot \left[1 - \left(\frac{\rho}{\rho_j}\right)^a\right]
\]

\[
v(\rho) = v_f \cdot \left[1 - \left(\frac{\rho}{\rho_j}\right)^{a+0.5}\right]
\]

For each of model the calculation of the above statistic fit are given in the below table for each representative sample-cell.

<table>
<thead>
<tr>
<th>Sample (Cell)</th>
<th>1 (1.3)</th>
<th>2 (1.1)</th>
<th>3 (1.0)</th>
<th>7 (2.1)</th>
<th>8 (2.0)</th>
<th>10 (3.1)</th>
<th>11 (3.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>R-square</strong></td>
<td>0.2781</td>
<td>0.9861</td>
<td>0.0755</td>
<td>0.1898</td>
<td>0.2620</td>
<td>0.2228</td>
<td>0.0947</td>
</tr>
<tr>
<td><strong>Adj R-sq</strong></td>
<td>0.2781</td>
<td>0.9861</td>
<td>0.0755</td>
<td>0.1898</td>
<td>0.2620</td>
<td>0.2228</td>
<td>0.0947</td>
</tr>
</tbody>
</table>

**Greenshields Model:**

\[
v(\rho) = v_f \cdot \left[1 - \left(\frac{\rho}{\rho_j}\right)\right]
\]

<table>
<thead>
<tr>
<th>Sample (Cell)</th>
<th>1 (1.3)</th>
<th>2 (1.1)</th>
<th>3 (1.0)</th>
<th>7 (2.1)</th>
<th>8 (2.0)</th>
<th>10 (3.1)</th>
<th>11 (3.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>R-square</strong></td>
<td>0.1559</td>
<td>/</td>
<td>-0.1814</td>
<td>0.0705</td>
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<td><strong>Adj R-sq</strong></td>
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<td>-0.5999</td>
</tr>
</tbody>
</table>

**Greenberg Model:**

\[
v(\rho) = a \cdot \log \left(\frac{\rho}{\rho_j}\right)
\]

<table>
<thead>
<tr>
<th>Sample (Cell)</th>
<th>1 (1.3)</th>
<th>2 (1.1)</th>
<th>3 (1.0)</th>
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<th>8 (2.0)</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>R-square</strong></td>
<td>0.2617</td>
<td>0.1887</td>
<td>0.0449</td>
<td>0.2005</td>
<td>0.0043</td>
<td>0.2955</td>
<td>-0.2408</td>
</tr>
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<td>-0.2408</td>
</tr>
<tr>
<td><strong>RMSE</strong></td>
<td>4.7327</td>
<td>12.9021</td>
<td>9.1329</td>
<td>6.8920</td>
<td>5.2879</td>
<td>8.7429</td>
<td>4.2665</td>
</tr>
</tbody>
</table>

**Pipes Munjal Model:**

\[
v(\rho) = v_f \cdot \left[1 - \left(\frac{\rho}{\rho_j}\right)^a\right]
\]

<table>
<thead>
<tr>
<th>Sample (Cell)</th>
<th>1 (1.3)</th>
<th>2 (1.1)</th>
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<td>5.2879</td>
<td>8.7429</td>
<td>4.2665</td>
</tr>
</tbody>
</table>

**Drew Model:**

\[
v(\rho) = v_f \cdot \left[1 - \left(\frac{\rho}{\rho_j}\right)^{a+0.5}\right]
\]

<table>
<thead>
<tr>
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<th>3 (1.0)</th>
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<td>6.8920</td>
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<td>8.7429</td>
<td>4.2665</td>
</tr>
</tbody>
</table>

Source: [Original from Author]
7.2. Comparative analysis of Inter-Cell Flows and Outflow as function of velocity

In the seventh chapter are given comparative analysis between updated CTM inter-cell flows and the updated CTM outflows as a function of velocities. It can be noted that the values of flows leaving an upstream cell and flow entering to e downstream cell is depended only by a single value of velocity, that is free flow velocity. An analogy between the inter-cell flows obtained by the only value of velocity and the flows as function of velocity can be performed on a well formulated CTM model. Prior to the realization of this analogy an analysis of the usual CTM fundamental diagram and variable velocity diagram is required.

![Variable velocity diagram](image)

Fig. 31. Variable velocity diagram of relationship of density
Source: Original from author

As can be seen from (fig 31), only the free flow speed or backward speed is implied, while VVD diagram contains the variable velocity in function of densities updates by CTM. Flows on the left diagram are calculated by $v_f \rho_{i-1}$, for $\rho \leq \rho_{cr}$, and by $w_i (\rho_j - \rho_i)$ for $\rho \geq \rho_{cr}$, while on the right diagram, flows corresponding to lower density values are similarly calculated by $v_f \cdot \rho_{i-1}$, for $\rho \leq \rho_{cr}$, but for every density value higher than $\rho_{cr}$, is implicated respective velocity values, $v_i$. As the computed p-values: 0.144, 0.525, 0.291 and 0.753 for cells 1.1, 2.1, 3.1 and 1.9 respectively, are greater than the significance level alpha 0.05 null Hypothesis $H_0$ cannot be rejected, can be concluded that all samples of inter-cell flows follow and flows calculated by updated velocity come from the same populations or follow same distributions.

8. INTEGRATION OF FILTER INTO NEW MODEL

The primary purpose of creating such a discrete model of a small part is not just the observation of traffic conditions and the observation of the traffic parameters relationship lows. At least this new model adapted to model and prescribe traffic conditions provides the availability of posing parameters in short term terms of time and distance and these advantages should be used for traffic management purposes. When it comes to control of traffic under real conditions, many well-known strategies today use the density data for calculation of the responsive parameters of signal plans such as duration of green time in signal.

Agreeing that specifically updating the density in certain segments in the upstream to signalized intersections is what makes the CTM model useful, we should think about integrating techniques that improve the model and tie up with real conditions. An important invention technique in the field of traffic state estimation is considered the Kalman Filter (KF) in 1960.
In this thesis the KF is used for traffic density prediction, and the term estimation is sometimes applied for referring to the density prediction by using filter techniques in new CTM model. In fact KF is a recursive algorithm that uses only the previous time-step’s prediction with the current measurement in order to make an estimate for the current state. This means the KF does not require previous data to be stored or reprocessed with new measurements. At every iteration, the KF minimizes the variance of the estimation error, making it an optimal estimator if linear and Gaussian conditions are satisfied. The KF works by making a prediction of the future and comparing the estimate with the measurements. Along with the prediction, an error covariance is calculated. KF consists of two phases: prediction or estimate phase and corrector or update phase, fig. 32. During review and application of KF in this dissertation, we will name these phases with terms of prediction and correction phase. Each of the phases is given by some sets of equations.

**Prediction Phase** contains calculation of predicted state vector $x_t$ and the state error covariance matrix $P_t$.

$$x_t = Ax_{t-1} + Bu_t + w$$

$$P_t = AP_{t-1}A^T + Q_{t-1}$$

$P_t$ represents the error covariance of state and is depended on the previous error covariance of state, $P_{t-1}$.

**Correction Phase** consists of equations:

$$K_t = \frac{P_tH_t^T}{H_tP_tH_t^T + R}$$

$$\tilde{x}_t = x_t + K_t(z_t - H_t\tilde{x}_t)$$

$$\tilde{P}_t = (I - K_tH_t)P_t$$

Fig. 32. Iteration process of Kalman Filter
During the simulation time, outflow takes zero values due to signal status-red light. Our purpose is also overview of the modeled outflow and the measured outflow. In fig. 33 are presented the graphic results of the KF modeled outflow of Cell 11 and measured outflow. Excepting the graphic results, the known statistic measure MAPE (8.10) (mean absolute percentage error) indicates a match of these values therefore the KF model is verified as suitable for traffic density prediction.

\[
MAPE = \frac{100}{n} \sum \left( \frac{|\rho_{\text{meas}(i)} - \rho_{\text{KF}(i)}|}{\rho_{\text{meas}(i)}} \right)
\]

\( n \) - the number of data, and in this case is the number of simulation time steps, 
\( \rho_{\text{meas}} \) - Measured density value of Cell i, and 
\( \rho_{\text{KF}} \) - KF density value of Cell i

The MAPE for the measured and modeled outflow is about 0 value, and for the results of predicted and measured densities is 0.166, 0.173 and 0.165 for cells 1.10, 1.9 and 1.6.

<table>
<thead>
<tr>
<th>Table 6 MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured and KF outflow of cell 1.1</td>
</tr>
<tr>
<td>MAPE</td>
</tr>
</tbody>
</table>

Following are given the graphical results of Kalman Filtering, performed with Mat Lab software.

Fig.33 Measured and KF outflow of Cell 1.1
Source: New CTM Model, MATLAB/R2015b

Fig.34 Measured and KF density of Cell 1.9
Source: New CTM Model, MATLAB/R2015b
As stated at objectives unit the main purpose of this dissertation is to develop a new model based on Cell Transmission Model-CTM that is capable to model traffic states and operations of urban arterials. As conclusion, the contribution of this dissertation is twofold:

- Improvement of the CTM models in order to be capable for modeling the instant traffic changes in urban segments and
- Review of filtering techniques for improving the new CTM and make it usable in traffic state prediction in real-time traffic control.

Through passing on the necessary steps toward fulfilling the first objective is shed light on the disadvantages and flaws of the existing CTM models so far. CTM was used mostly to model one way traffic networks, with simple composition of the connections as freeway, with or without on ramping and off ramping segments. Obvious limitations are overcome in our model by taking in considerations:

- Physic features/geometric features: Complex composition of urban segment approaching to intersections, providing merge and diverge cells. Basic principles (the flow from cell i-1 to cell i is the smallest value between the sending flow from the upstream cell or receiving flow of the downstream cell and capacity flow) of the CTM models such fully adapted to the merge and diverge configurations.
- Dynamic traffic of urban segments: In the original CTM, besides the special node complexities regarding to diverging and merging, was neglected the definition of out flow
from the nearby stop-bar cell (SBC). As it is known, by the traffic flow nature on the
signalized intersections, at the beginning of green interval the discharge rate is low, with
higher discharge headways do to the reaction time and start up time of the first vehicle, which
increases to the capacity flow until the queue is being dissipated. Implication of new demand
function makes new CTM model capable emphasize the difference of the outflow rate from
the SBC on different consequential portions of green interval. Some sending and receiving
schemes of the new demand function were reviewed even beyond the requirements set forth
for the development of a concrete model of the dissertation.

- Detailed characterization of traffic parameters of fundamental diagram FDR for every single
cell as a requirement for calibration of CTM is the novelty of this dissertation while in the
simple CTM models free flow speed, critical density, jam density, capacity flow and
backward speed had a single value for the entire segment. The analyzed parameters can be
considered as microscopic since are evaluated for each individual entity/vehicle as a
component of traffic. The length of road cells are chosen of 25 meters long while.

**Results of the setup example**

Case study is designed to evaluate the performance of original CTM and new proposed CTM
model in terms of predicting the traffic flows, densities under some different scenarios as presented
in the section below. A test bed- urban arterial is chosen to model the proposed CTM on it. Model
is compiled within C# programming environment algorithm. After the simulation results are
obtained, a comparative analysis of CTM model and proposed model of this dissertation with real
traffic measurement is given with the explanation of the accuracy improvements and disadvantages
on usefulness and implementation on future complex urban areas modelling and their problem
solving. In simulation is used cycle length of subjected intersections is 120 seconds, which means
that one of them comprises of 52 time steps, the overall number of cycles within the simulation is
7.5 cycles, consequently the number of time steps in simulation run is 390.

- Results of initialization that means the initial conditions of the whole artery, when all the
cells are empty (density is zero), but the entry cell of the first segment (Cell 1.10) feeds the
artery with average traffic flow until is get a stable state of traffic. A special survey is
dedicated to the most representative cells i.e. initial cells on the segment (Cell 1.8, Cell 2.8
and Cell 3.8), cells upstream to intersections (Cell 1.1, Cell 2.1 and Cell 3.1) during the first
cycle, consequently during first 52 time-steps. There is a faster increase of density (fill up
with vehicles) of the cells of the first segment, that start immediately after the second time-
step , while there is an obvious delayed increasing of density on the cells of the far away
segments . The earlier increasing of density is noted by after the tenth time-step to Cell 2.8,
which reaches the average value in the middle of cycle (around time-step 25). The most
delayed increasing of density is noted to the most far away cell (Cell 2.1) of the segment 2.
It is important to note that the high density values are never reached during the first cycle,
by the last segment that it is understood that a single cycle cannot be considered as a
“feeding” or “warm up” period during which the artery sufficiently fills up. The Cell 3.1
positioned upstream to intersection 3, starts to fill up with vehicles by the end of cycle, at
time-step 36. Comprehensively the model can satisfactorily model the traffic states on initial
phases.

- Results on left turn lanes: For this survey is chosen left turn lane of segment 1 that is covered
by Cell 1.2L and Cell 1.1L. Green interval starts from the time-step 23 and lasts till to the
time steps 33. Regarding to the outflow of these two cells during the red interval, we can
analyze the first and third diagrams (first Time-Step Interval from 0-23 and second Time-Step Interval from 33-52). It can be noted that for Cell 1.2L, during the first red interval in cycle flow and density increases proportionally while in the second red interval in cycle have and decreasing trend. Nevertheless it can be noted the flow from the Cell 1.1L is zero during both red intervals of the cycle, moreover the density tends to increase on the second red interval. This can be interpreted that even though no vehicle is released from Cell 1.1L during red interval, this cell continues to receive vehicles from its previous Cell 1.2L.

- Model can describe light traffic, medium and congested traffic conditions. In light traffic conditions, in great mass traffic flow is dictated by the traffic signal status, consequently we have a harmonious move of the density curve that follows the signal status. An increasing trend of density during green intervals and in opposite and a decreasing trend of density during green interval for each cell is noted. Unlike the light conditions, in medium congestion is obtained a rapid increasing of the density to each cell. Fluctuations of the density from the lowest to the highest values are obvious during the first three cycles to all cells of the artery. The lowest is zero and the highest takes values till 0.4veh/m, 0.5veh/m or 0.6veh/m, depending on the number of lanes that cell covers. After the first three cycles, density never get back to the lowest values but their fluctuations are between 0.4veh/m to 0.6veh/m.

Beside the possibility to update the densities of cells through time step, the new CTM model offers the estimation of evolution of the velocities which may be an important factor of challenging real time prediction and control strategies. The potential to obtain velocity values is discussed for different velocity/density relationship models and showed to be adaptable for the newest ones, such as Drew and Pipes Munjal Model.

**Second achieved objective** is usable of model for predicting traffic parameters at short time intervals (time steps) and short length distances (cells) as part of the segments between the signalized intersections. At least this new model adapted to model and prescribe traffic conditions provides the availability of posing parameters in short term terms of time and distance and these advantages should be used for traffic management purposes. Today we are witness to high achievements in terms of sophistication of techniques for controlling traffic under real-time conditions. An important invention technique in the field of traffic state estimation is considered for integration on the model as it is the Kalman Filter (KF). KF is a recursive algorithm that uses only the previous time-step’s prediction with the current measurement in order to make an estimate for the current state. This means the KF does not require previous data to be stored or reprocessed with new measurements. At every iteration, the KF minimizes the variance of the estimation error, making it an optimal estimator if linear and Gaussian conditions are satisfied.

**Results of CTM-KF model**

A numerical example is provided for the sake of illustration of the calculation process of the KF in this unit. In calculation process of the KF are included formulation of equations -state transition matrices-and state vector. For the sake of illustration of the KF process, the first arterial segment modeled in CTM is chosen for KF estimation. It is comprised of eight ordinary cells two diverge cells and three cells for left and right turn. The cell length is kept same as well as the time step duration, so \( L=25\text{ m} \) and \( T=2.3\text{ sec} \) to fulfill the known CTM condition that \( L/\nu_f \geq T \). An important issue is the definition of the model state and measurements. Since the aim is to estimate the densities, into our transition model the state vector contains the density parameters of the all
above mentioned thirteen cells. The essence of the KF is to correct the state parameters, densities in this case, by usage of the measurements or observations of any parameter that can be easily obtained. At this example the exiting flow (outflow) from the cell nearby to stop bar is chosen as measurement. Measurements are collected by sensors-pavement sensors –detectors, in online application of estimated parameters for traffic control. The MAPE for the measured and modeled outflow shown to have an accuracy level for modeled values and measured values of flows and density.